IRSN
INSTITUT
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Enhancing nuclear safety

Uncertainties and sensitivities study methods applied to the dynamic fuel cycle

*Example of **MORRIS** and **FAST** methods*

Technical Workshop in Fuel Cycle Simulation

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Overview

- Issues
- MORRIS method
  - Introduction & Goal
  - Principle
  - Sensitivity evaluations
  - Application case
- Sobol' indices - FAST method
  - Introduction & Goal
  - Application case
- Conclusions

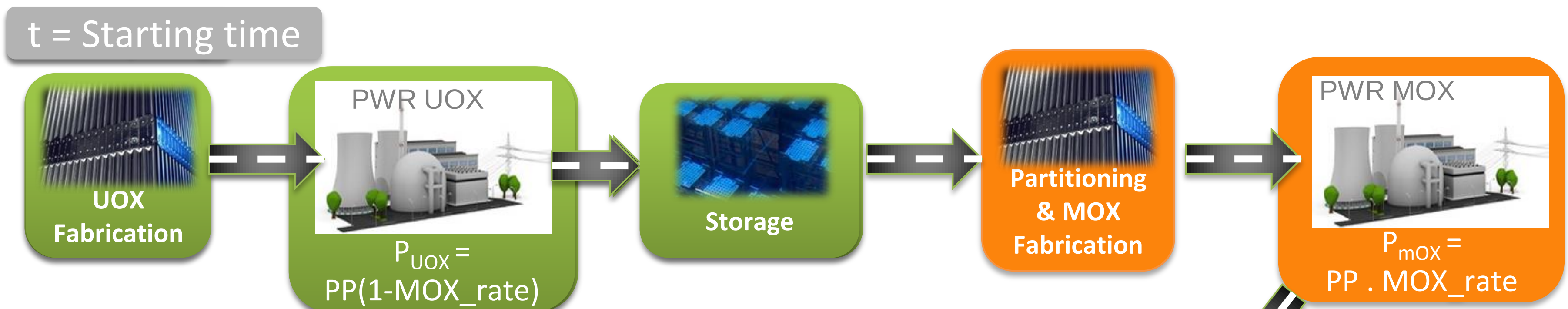
Issues

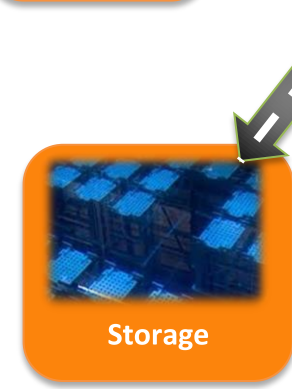
Large number of hypothesis

- Reactors: type, starting time, power…
- Fuel cycle options: open or recycling, enrichment, Burn Up to be achieved…

Parameters values are not exactly known → Possibility to define the variations ranges

Example





Parameters:

- BU UOX and MOX [30 , 60] GW.j/t
- PWRs loading factors [0.65 , 0.85]
- Starting PWR MOX time [30 , 50] years
- MOX fraction [0.05 , 0.15]
- Minimum cooling time [4 , 20] years
- MOX fabrication time [1 , 3] years

Issues

Large number of output

- Inventories: needs on natural U or Th, in reactor, facility, stock…
- During the time, at a specific date, at the end…
- No a priori to the parameters effects on the output

What is the impact of the parameters uncertainties on the output?

Output sensitivities depend on

- the shape of the studied output
- the defined parameters
- their variations ranges

Which methods apply to perform Sensitivities study?

MORRIS method

How to evaluate the *cross* “weight” of interactive input parameters?

MORRIS method

Introduction & Goal

- Determinist approach (qualitative study)
- Screening method
- Don't require a lot of calculations

Adapted to the preliminary study

- Goals
  - identify non-influential inputs
    → Can be fixed
  - identify and prioritize the most influential inputs
    → Select the mains parameters
  - Identify the linear effect of the input
    → Can be studied separately
  - Identify the non-linear and/or interaction effects of the input
    → Must be studied together

Reduce the inputs number to perform a precise study

MORRIS method

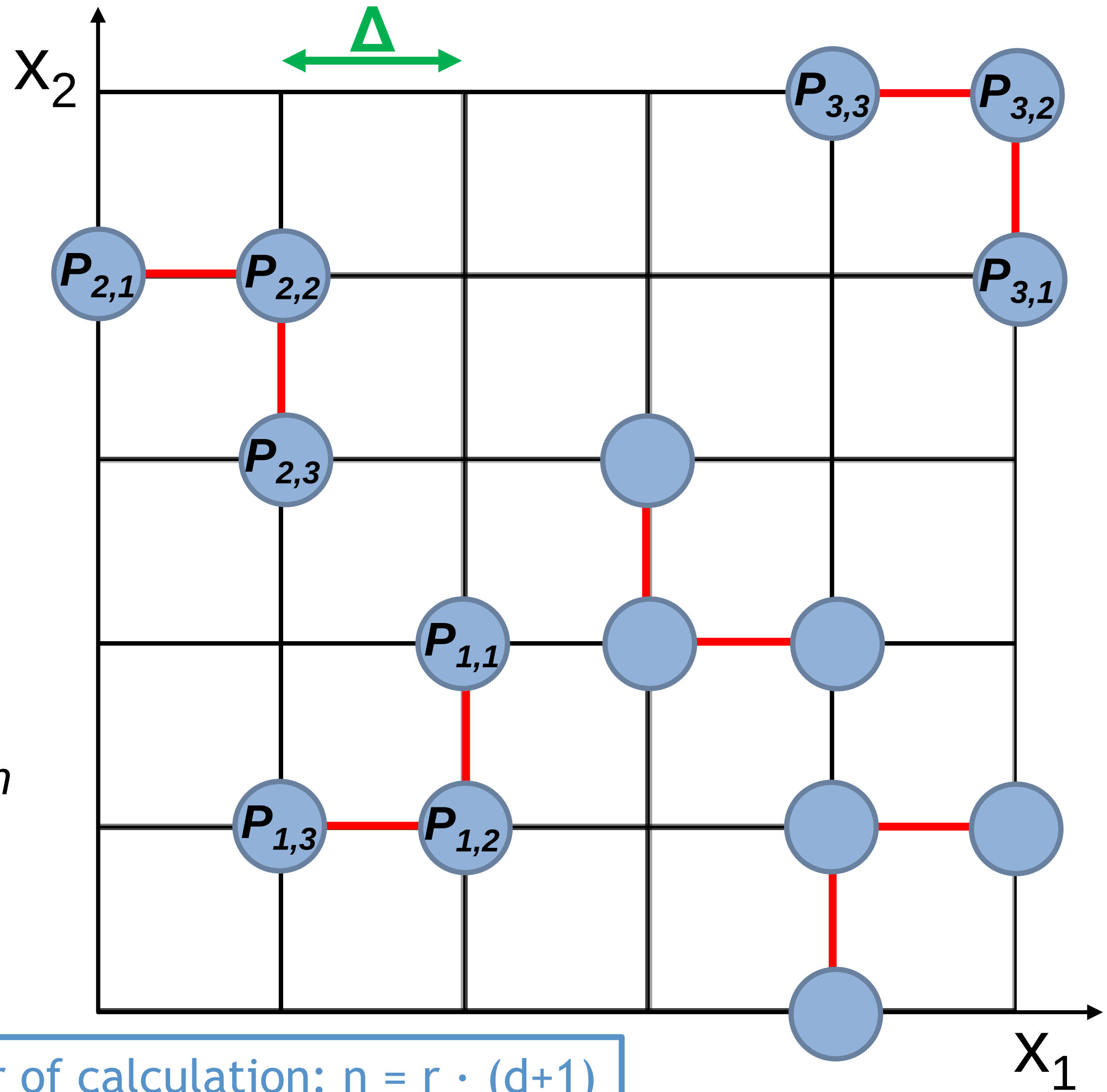
Principle

Example

- x_1, and x_2 parameters
- $f(x_1, x_2)$ observable

Method

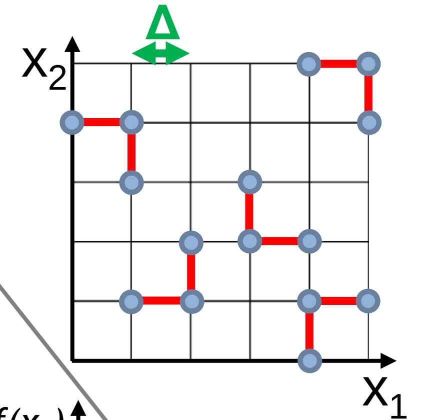
1. ***Discretization*** *of the inputs space (d = dimensional grid)*
2. ***Sample One At a Time*** *(d +1 points)*
   - *Base point* ($P_{1,1}$)
   - *Sample other points of the group* ($P_{1,k}$) *by sequential bifurcation method (random permutation)*
3. ***Repeat r times*** *the step 2 by method of space-filling optimization*



Total number of calculation: $n = r \cdot (d+1)$

Recommended $r \in [4,10]$

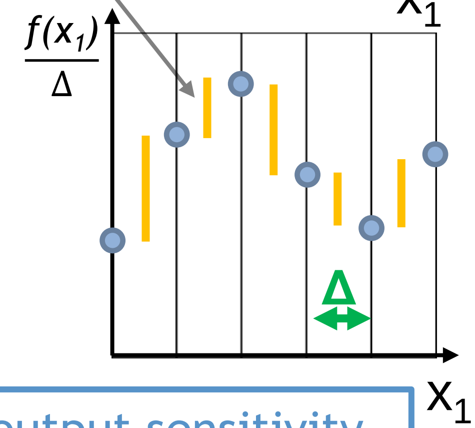
Sensitivity evaluations



Elementary effect

$$E^i{}_x = \frac{f^i(x_1 \pm \Delta) - f^i(x_1)}{\Delta}$$

Example: $E^1{}_{x1} = \frac{f(P_{1,3}) - f(P_{1,2})}{\Delta}$



Measure of x_j influence

$$\mu_{xj}{}^* = \frac{1}{r}\sum_{i=1}^{r}\left|E^i{}_{xj}\right|$$

Evaluate the average of the absolute value of the output dispersion produced by x_j

~ Average length of the yellow lines

More $\mu_{xj}{}^*$ is large more the parameter contributes on the output sensitivity

Measure of non-linear and/or interaction effects

$$\sigma_{xj} = \sqrt{\frac{1}{r}\sum_{i=1}^{r}\left(E^i{}_{xj} - \frac{1}{r}\sum_{i=1}^{r}E^i{}_{xj}\right)^2}$$

Standard deviation of the elementary effect

If σ_{xj} is small (effects are almost constant)
~ linear effect
Else ~ non-linear and/or interaction effects

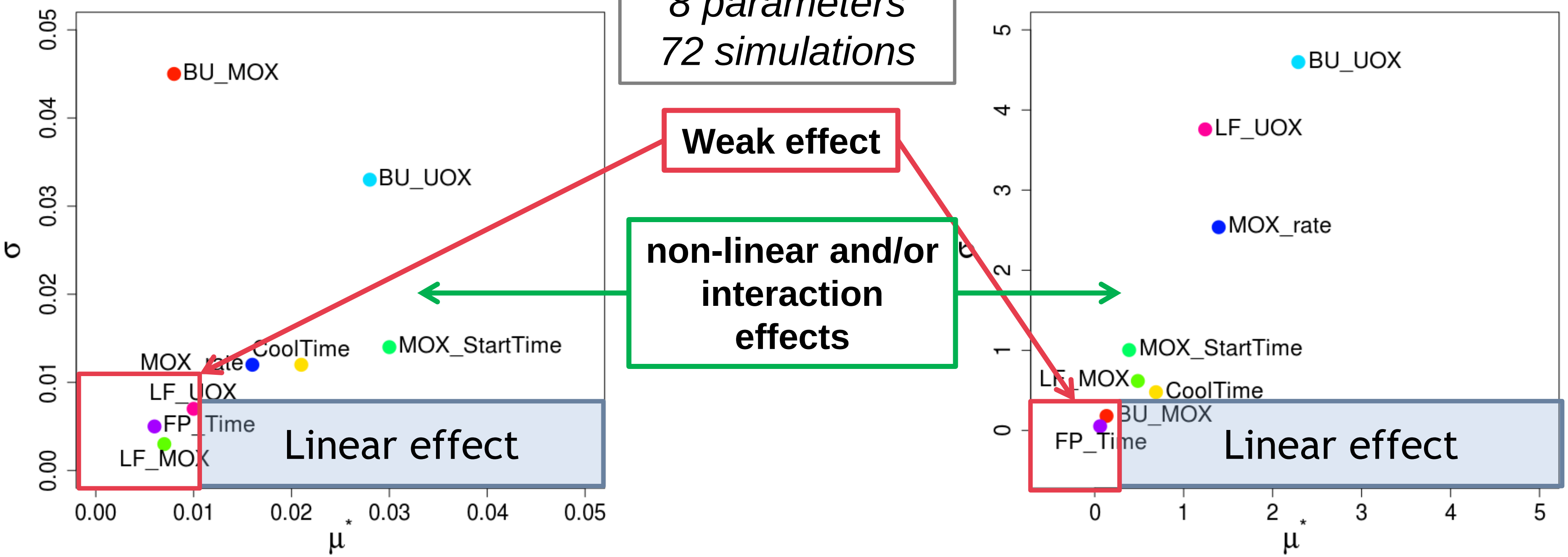
MORRIS method

Application case

Maximal value of the Pu content in MOX fabrication facility

Pu inventory at the end of the scenario

8 parameters
72 simulations



Weak effect : *LF_MOX, LF_UOX, FP_Time*
Non negligible effect: *BU_MOX, BU_UOX, MOX_StartTime, CoolTime, MOX_rate*

→ ***5 parameters***

Weak effect : *FP_Time, BU_MOX*
Non negligible effect: *BU_UOX, LF_UOX, MOX_rate, MOX_StartTime, LF_MOX, CoolTime*

→ ***6 parameters***

Number of parameters are reduced for sensitivity study with height calculation cost

Sobol' indices - FAST method

Beyond these deterministic effects ?

How much each input variable uncertainty contributes to the output uncertainty?

(Another "sensitivity" measure)

Sobol' indices - FAST method

Introduction & Goal

- Probabilistic approach *(based on probability of density law)*
- Sobol' indices ~ variance based sensitivity indices (quantitative study)
- Indices:
  - Number of indices : $2^d - 1$
  - Mains indices: total S_{Ti} and 1th order S_i effects of the parameter i

$$S_i = \frac{Var[\ E[f(\boldsymbol{x})/x_i]\ ]}{Var[f(\boldsymbol{x})]} \qquad S_{Ti} = S_i + \sum_{j \neq i} S_{ij} + \sum_{j \neq i, k \neq i, j<k} S_{ijk} + \cdots$$

- Interpretation:
  - If S_i and S_{Ti} are large → Important sensitivity at the parameter
  - $S_i = S_{Ti}$ → Linear effect
  - $S_i \neq S_{Ti}$ → Interaction effects

- Sobol' indices calculations:
  - Standard method → NS >> (d+2) · 10 000
  - FAST method (Fourier amplitude sensitivity test).

→ d · 100 < NS < d · 1000
But it may be unstable and biased when the number of inputs increases (d ~ 10)

Sobol' indices – FAST method

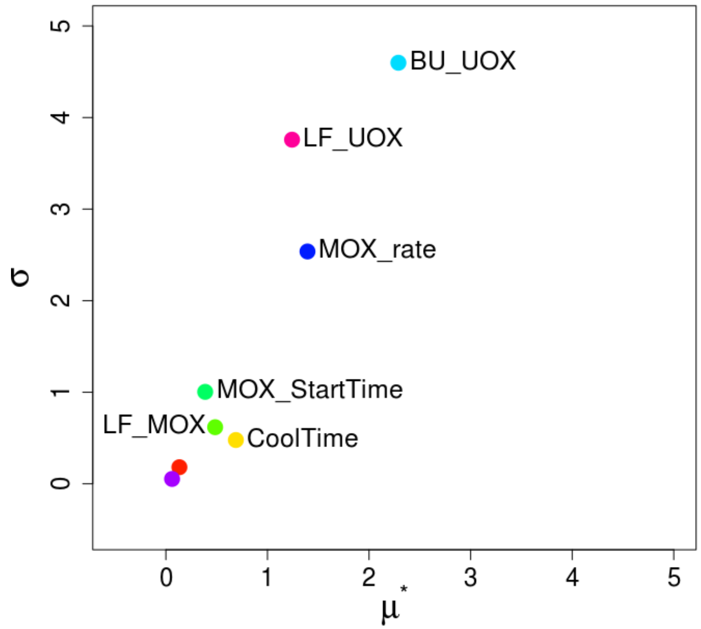
Application case

Pu inventory at the end of the scenario

Sobol' indices calculated with FAST method only for the parameters selected with the MORRIS method

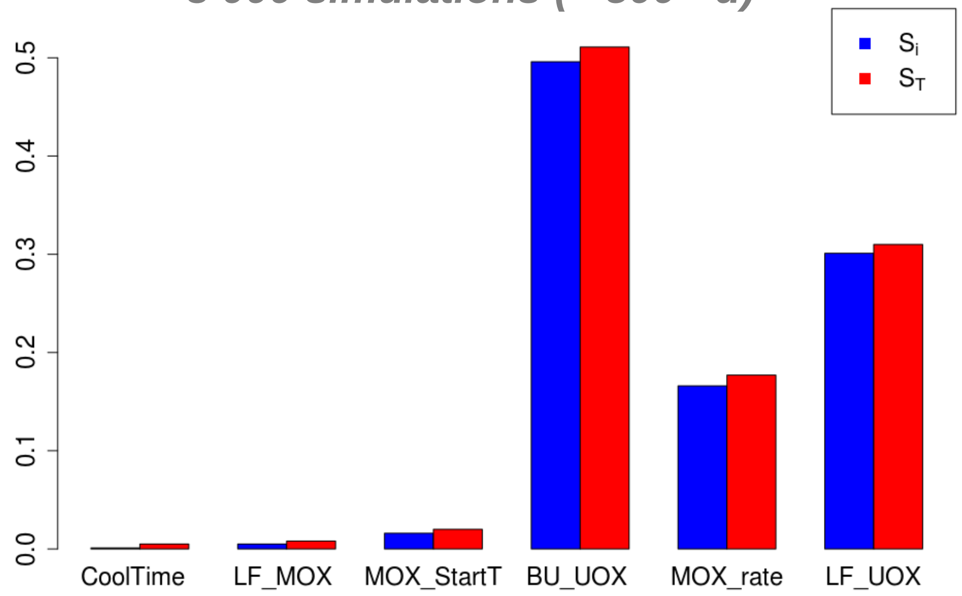
MORRIS result

72 simulations



FAST result

3 000 simulations (= 500 · d)



Weak effect : *MOX_StartTime, LF_MOX, CoolTime*
Main parameters: *BU_UOX, LF_UOX, MOX_rate*

Sum of Sobol' indice: $\sum S_{ij} = 0{,}985$

and $\sum S_T = 1{,}031$

Confirmation of the main parameters

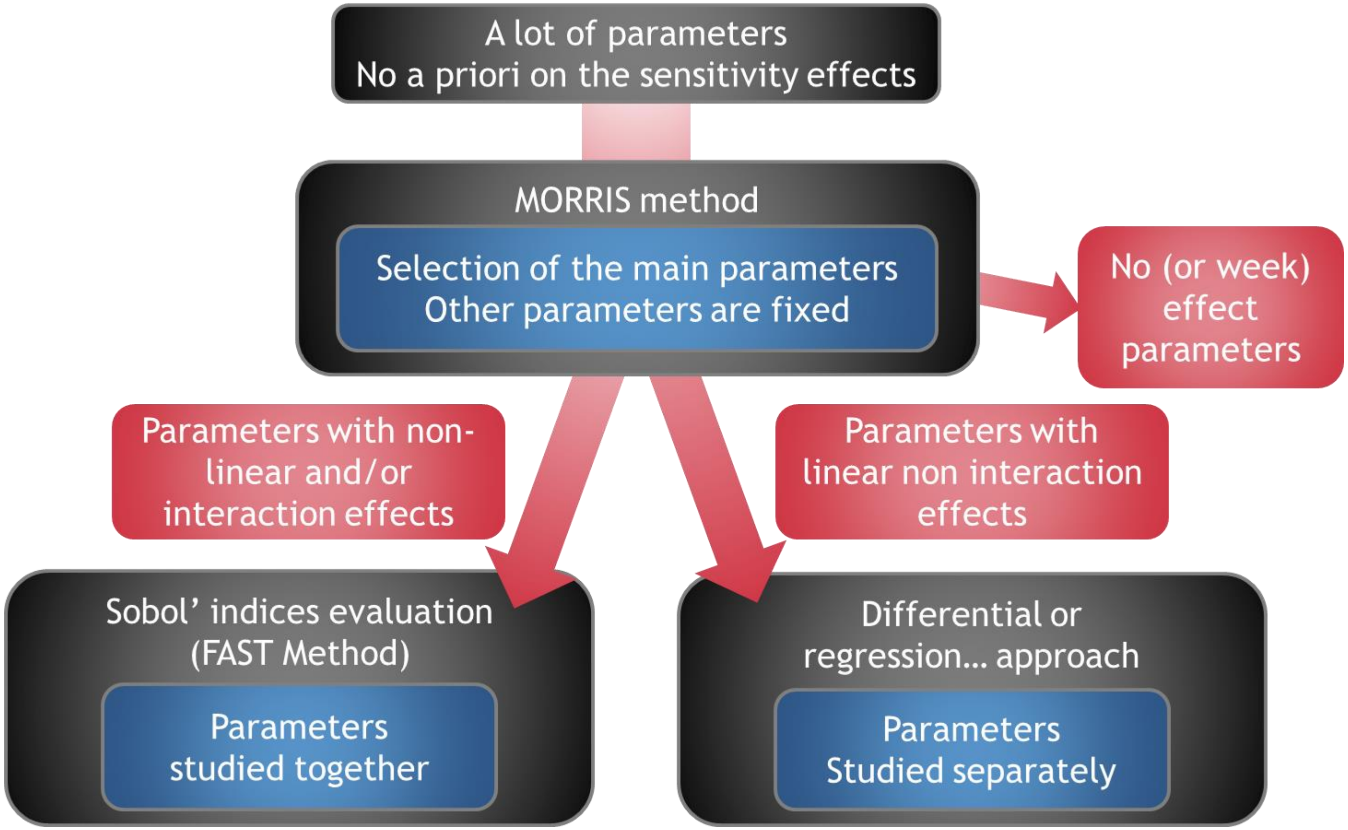
Estimation of the parameter impacts

Small part of interacting effect

Conclusions

- The fuel cycle **scenarios** studies include **a lot of parameters**
- Evaluate precise **sensitivity** require **huge number of calculations**
- **MORRIS method** can help
  - to **select** the mains parameters
  - to **separate** the parameters with linear effect and the others (non-linear and/or interaction effects)
  - by consequence to **reduce** the number of parameters for a precise sensitivity study
- **Sobol' indices** can help
  - to **Estimate importance** of a parameter on the sensitivity
  - it require **huge number of calculations**
  - **FAST method** can be used to **reduce** the number of calculations

Conclusions



Thank you for your attention

Interesting reference:

B. Iooss and P. Lemaître. "A review on global sensitivity analysis methods". HAL Id: hal-00975701 https://hal.archives-ouvertes.fr/hal-00975701. 2014

Thank you for your attention

Questions?

Calculation packages at IRSN/SNC

CLASS (Core Library for Advanced Scenario Simulation)

- **Dynamic fuel cycle simulation tool**
- **Open source package of C++ libraries using ROOT libraries**

https://root.cern.ch/

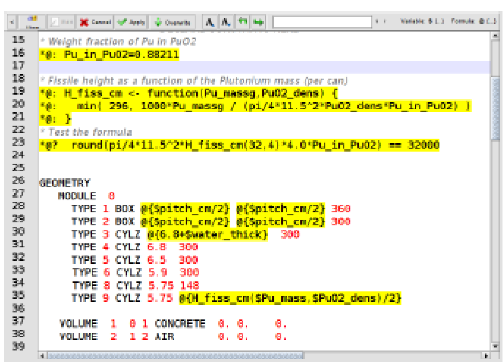
- **Collaborative development (CNRS and IRSN)**
- **Presented in details in the previous talks:**
  - “Reactor model in CLASS”. B. LENIAU
  - “Pu multi-recycling in PWR”. F. COURTIN
  - “Am mono-recycling using PWR - a waiting strategy”. A-A. ZAKARI-ISSOUFOU

Calculation packages at IRSN/SNC

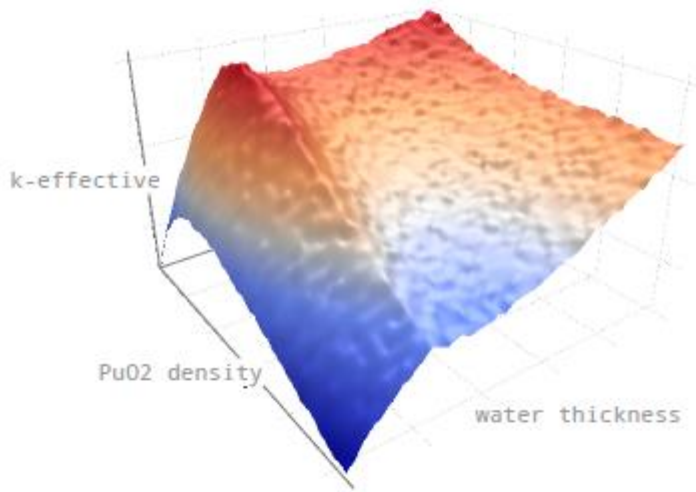
PROMETHEE

http://promethee.irsn.org

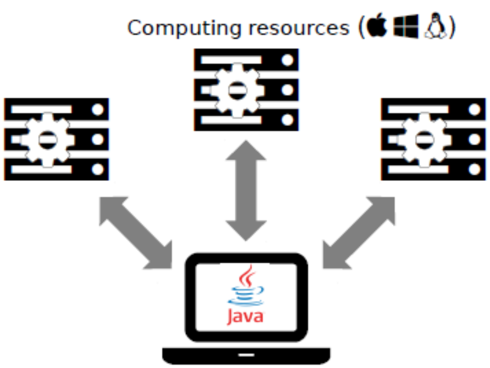
- **Generic front-end dedicated to parametric studies** (Editor integrated)
- **Parallel distribution of calculations relies on its cross-platform back-end** (any kind of computing resources are compatible: clusters, workstations…)
- **Plugins available with IRSN codes, MCNP, CLASS…** (new code plugin ~ working day)
- **Extensible architecture to plug algorithms for advanced engineering based on R language** (response surface, uncertainties propagation, optimization, calibration, inversion)



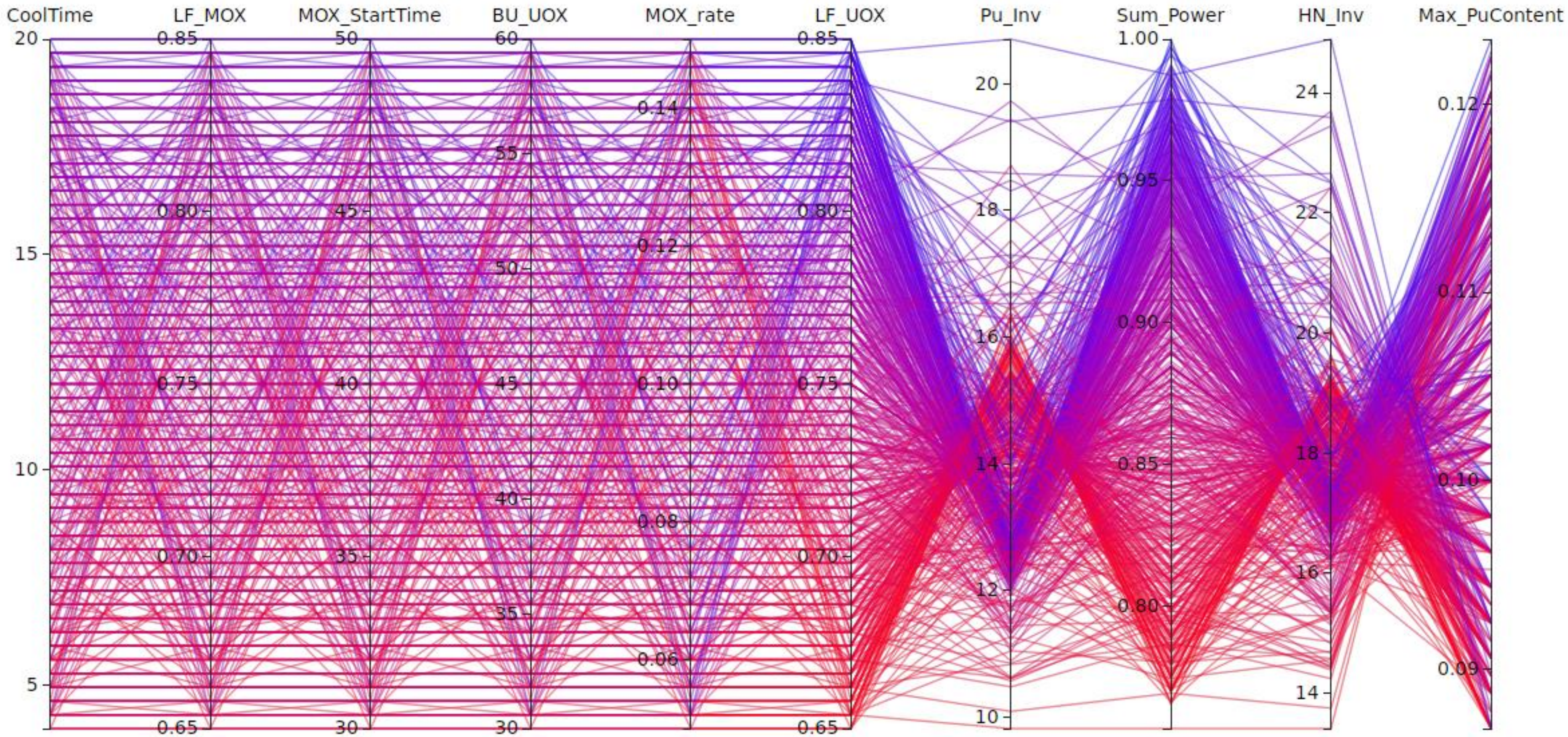
Input text file editor with parameters



Response surface model from MORET neutron simulation software



Application case



Sensitivity study methods

Number of simulations = NS
Number of parameters (dimension) = d

- Using metamodels
  - generalized linear model → The precision of the sensitivity depend on the precision of the meta model
  - kriging
  - neural network → Depend on the : NS & variability of the results
- Screening
  - Factorial plan → $NS \sim 2^d$
  - MORRIS method → $4 \cdot (d+1) < NS < 10 \cdot (d+1)$
- Sobol' indices calculations:
  - Method standart → $NS >> (d+2) \cdot 10\,000$
  - FAST method → $d \cdot 100 < NS < d \cdot 1000$

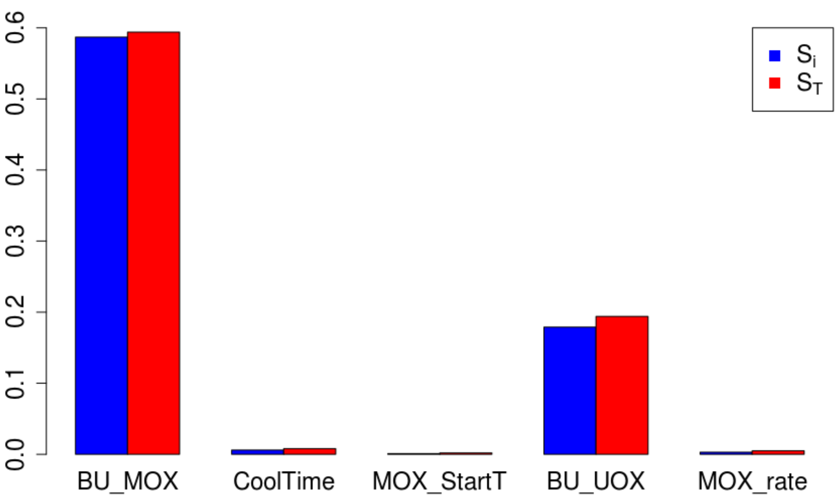
Sobol' indices - FAST method

Application case

Sobol' indicies calculated with FAST method only for the parameters select with the MORRIS method

- **Maximal value of the Pu content in MOX fabrication facility**

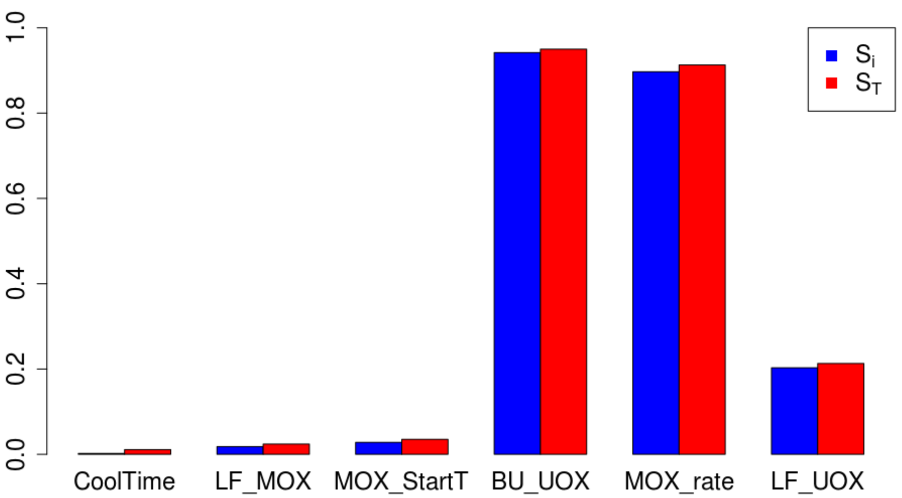
500 simulations



Weak effect :*MOX_StartTime, CoolTime, MOX_rate*
Main parameters: *BU_MOX, BU_UOX*
Small part on interacting effect

- **Pu inventory at the end of the scenario**

600 simulations



Weak effect : *MOX_StartTime, LF_MOX, CoolTime*
Main parameters: *BU_UOX, LF_UOX, MOX_rate*
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Sobol' indices - FAST method

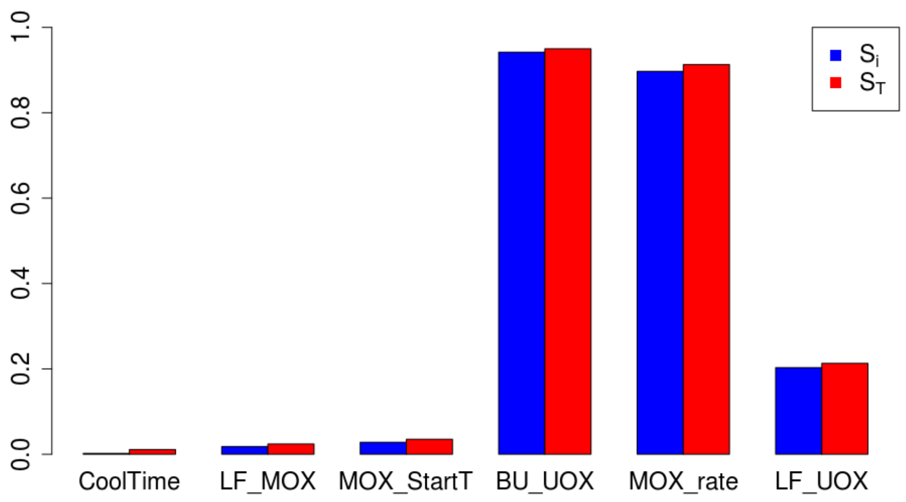
Application case

Sobol' indices calculated with FAST method

- **Pu inventory at the end of the scenario**

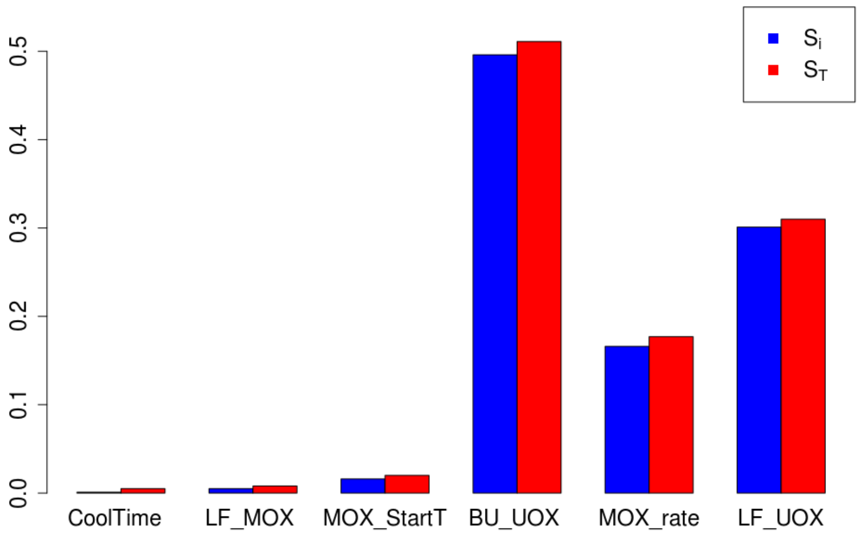
FAST result

600 simulations (= 100 · d)



FAST result

3 000 simulations (= 500 · d)



Careful to perform enough calculations